PERIODIC GROUPS WHOSE ELEMENT ORDERS ARE SMALL

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A group G is said to be *periodic* if, for every $g \in G$ there exists a natural n such that $g^n = 1$. If there exists a common n with $g^n = 1$ for all $g \in G$ then n is called a *period* of G and the smallest such n is said to be the *exponent* of G. A group G is *locally finite* if every finite set of its elements is contained in a finite subgroup. The class C_n of groups of period n is a *variety*, i.e. it is closed under taking subgroups, factor groups and cartesian products.

The set $\omega(G)$ consisting of all orders of elements of G is called the *spectrum* of G. If $\omega(G)$ is finite then $\mu(G)$ is the set of maximal elements of $\omega(G)$ with respect to division.

We consider the following general question. If $\omega(G)$ is given, what can we say about G, in particular, is such G locally finite? Some recent results here are the following:

- (E. Jabara, D. V. Lytkina, V. D. Mazurov, A. S. Mamontov, 2014) Suppose that $\mu(G) = \{4, 5, 6\}$. Then G is locally finite and one of the following statements holds:
    1. $N = O_5(G)$ is a non-trivial elementary Abelian group, $G = NC$, where C is isomorphic to $\langle x, y \mid x^3 = y^4 = 1, x^y = x^{-1}\rangle$, or $SL_2(3)$, and C acts freely on N.
    2. $T = O_2(G)$ is a non-trivial elementary Abelian group, and $G/T \simeq A_5$.
    3. G is isomorphic to S_5 or S_6.
- (E. Jabara A. S. Mamontov, 2015) Suppose that $\mu(G) = \mu(L_3(4)) = \{3, 4, 5, 7\}$ then $G \simeq L_3(4)$.
- (A. S. Mamontov, 2015) If $\mu(G) = \{6, 7\}$, then G is an extension of a locally finite group by a group of odd order.